Math 312

Integral Theorem Questions

1. Evaluate $\int_C 2xy^3dx + 3x^2dy$ where C consists of the perimeter of the unit square with vertices at $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$ described in the anti-clockwise direction.

2. Verify Green's Theorem for $\oint_C 2xy^3dx + 3x^2dy$ where C is $y = x^3$ from $(0,0)$ to $(1,1)$ and $y = x$ from $(1,1)$ to $(0,0)$.

3. Verfiy the divergence theorem for $\iint_S x^2z^2dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

4. Use Stokes' theorem to find $\oint_C 2ydx - zdy + ydz$ with C the intersection of the plane $2x + y + z = 0$ and the cylinder $x^2 + y^2 = 1$.

5. Verify the divergence theorem for $\mathbf{F} = (x, 2y^2, 3z^2)$ and $V : 0 \leq x^2 + y^2 \leq 9$, $0 \leq z \leq 1$.

6. Verify Stokes' theorem on the triangle with vertices $(2,0,0)$, $(0,2,0)$, $(0,0,2)$ and $\mathbf{F} = (x^2 + y^2, y^2, x^2 + z^2)$.

7. Calculate the surface integral $\iint_S \mathbf{v} \cdot \mathbf{n}dS$ where $\mathbf{v} = (x - z^2, 0, xz + 1)$ and S is the surface that encloses the volume $x^2 + y^2 + z^2 \leq 4$, $z \geq 0$.

   (a) directly,

   (b) by the divergence theorem.

8. Verify Stokes' theorem for $\mathbf{v} = (y^2 - xyz, 3x + 2x^2, x^3 + y)$ and C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = 25$ and the plane $z = -4$.

9. Verify Stokes' theorem for $\mathbf{F} = (y, -x, yz)$ over the part of the paraboloid $z = 2(x^2 + y^2)$ for which $z \leq \frac{1}{2}$.

10. A closed surface is made up of S_1 the part of the paraboloid $z = 1 - x^2 - y^2$ with $z \geq 0$, together with S_2, the circular disc $x^2 + y^2 \leq 1$, $z = 0$.
    Verify the Divergence theorem for this closed surface for $\mathbf{F} = (2x, 2y, 1)$.

11. Verify the theorem of Stokes for the curve cut on the plane $2x + y + z = 0$ by the cylinder $x^2 + y^2 = 2y$, and the vector $(2y, -z, y)$ the direction round C is positive when viewed from $(0,0,3)$.